REVIEWS

Statistical Theory of Irreversible Processes. By R. EISENSCHITZ. Oxford: Clarendon Press, 1958. 84 pp. 8*s*. 6*d*.

There is probably no physical generalization more firmly established than the Second Law of Thermodynamics. Indeed, the thermodynamic theory of equilibrium and its statistical basis are so well understood that there seems little room for further fundamental advance in this subject. Such, however, is far from true of the theory of passage towards equilibrium: here even the fundamental principles are still under dispute, and are the subject of much discussion at the present time.

It is generally agreed that a non-equilibrium situation, like a state of equilibrium, should be described by a probability distribution over phase space (or in quantum mechanics by the corresponding density operator). In equilibrium theory this distribution is obtained by maximizing the entropy, which is a well-defined integral of the distribution function. In non-equilibrium theory, however, this procedure will not work owing to the reversibility of the microscopic equations of motion; if the same integral is maximized at a certain instant, the resulting distribution function yields in general no instantaneous change in the macroscopic observables whose values are used to specify the non-equilibrium situation; indeed the entropy itself remains constant throughout time because of Liouville's theorem.

This difficulty was by-passed by Boltzmann in arriving at his famous equation for the time-development of the properties of a perfect gas in disequilibrium. The essence of Boltzmann's theory was to ignore the prior correlation of the velocities of two colliding molecules, so that the history of an individual molecule could be regarded as a stochastic time sequence. With this assumption Boltzmann was able to prove his famous H-Theorem, according to which the entropy per particle increases steadily with time. For strongly interacting systems, however, any statistical definition of the entropy must include particle correlation terms, if it is to agree with the thermodynamically measured entropy; thus Boltzmann's equation cannot be directly applied.

Historically, the next successful theory of an irreversible phenomenon was the Einstein theory of Brownian diffusion. A Brownian particle is supposed to be subject not only to viscous drag but also to a rapidly fluctuating force with a very short correlation time. In this way irreversible behaviour is obtained, but at the expense of introducing a somewhat artificial theoretical distinction between two effects which are actually of the same physical origin.

The most powerful result yet obtained in the theory of disequilibrium is undoubtedly that of Onsager, and has been justly described as the Fourth Law of Thermodynamics. By very general arguments, appealing only to microscopic reversibility and to the linear character of many dissipative phenomena, Onsager was able to show that in systems where two dissipative processes interact (such as diffusion and thermal conduction) there is a symmetry between the mutual effects of the two processes. Onsager's Law is of great generality and has been verified experimentally with the utmost precision in many different situations. It is, indeed, the most important single statement in the modern subject of non-equilibrium thermodynamics.

It is against this background that current work must be viewed. In non-equilibrium thermodynamics the entropy is calculated by integrating an entropy density, which in turn is estimated by assuming equilibrium conditions to prevail in each small element of the system. It is clear, therefore, that it is not the entropy but the entropy density which should be given a proper statistical definition, and Boltzmann's theory implicitly does this. But it might be asked whether one cannot, as Boltzmann did, obtain a satisfactory statistical theory which makes no explicit reference to the entropy. This is the line of thought which has been pursued most diligently by recent workers, and various procedures have been proposed. It is well recognized, for example, that although the distribution in phase space of a closed system develops reversibly, nevertheless, if this distribution is 'coarse-grained'—that is to say averaged over cells of small but finite phase volume—then the entropy calculated from the coarse-grained distribution increases steadily with the time, and from a macroscopic point of view the system shows irreversible behaviour. In practice the coarse-graining procedure is necessarily drastic; one must average over the co-ordinates and the momenta of all the particles except a very few so as to obtain an equation of motion in a reasonably small number of variables. Unfortunately, it then becomes very difficult to make any quantitative statement about the time-development of the few-particle distribution functions, so that although irreversible behaviour is ensured, the transport coefficients are still inaccessible to calculation.

A different but related procedure has been developed by Kirkwood and his collaborators. Kirkwood postulates the existence of a small finite time, short compared with the macroscopic relaxation time, but long enough for the average force on a molecule due to its neighbours to be effectively randomized. He then obtains equations for the transport coefficients of a liquid in terms of the autocorrelation integrals of the intermolecular forces and the problem is then reduced to evaluating these integrals. Though Kirkwood's formalism seems to be essentially correct its exploitation demands laborious analysis and has not yet proved very fruitful in the interpretation of experimental data.

Dr Eisenschitz's little book does not, unfortunately, do justice to its subject. The chapters appear to be arranged in an almost random order: the last (Chapter IX) is an elementary exposition of probability theory; Boltzmann's H-Theorem appears at the end of Chapter VII, which is concerned with statistical quantum mechanics; the theory of Brownian movement and of stochastic processes generally does not appear until Chapter VIII, and at the end of this chapter there is a superficial mention of Onsager's theorem, with no reference to its method of derivation. Equilibrium statistical mechanics is dealt with summarily in Chapter II, which occupies exactly four pages and concludes with the remarkable statement: 'These examples should be sufficient to show the achievements and limitations of the classical theory of equilibrium.' The meat of the book ought to be in Chapter III, entitled 'Irreversibility', but on page 11, after promising to show that a coarse-grained distribution develops irreversibly, the author introduces without warning the time-smoothing device, which has no logical connexion with the coarse-graining procedure. In this of all chapters the author should have attempted to make clear the necessary and sufficient conditions for irreversible behaviour, but he provides no indication as to what sort of phenomena ought to be irreversible, or why, and makes no comment on the question whether irreversibility can occur in a system containing a finite number of particles. In view of the author's scientific reputation, the book is a serious disappointment to anyone who wishes to understand clearly what has been achieved and what are the outstanding problems in this field.

H. C. LONGUET-HIGGINS

An Introduction to Fluid Dynamics. By G. TEMPLE. Oxford: Clarendon Press, 1958. 195 pp. 25*s*.

The chapter headings in this short book are: 1. Introduction; 2. The properties of the perfect fluid; 3. Bernoulli's theorem; 4. Irrotational flow; 5. The velocity potential; 6. The complex potential and velocity in plane parallel flow; 7. 'Point' sources, doublets, and vortices in plane parallel flow; 8. Equivalent layers of sources, doublets and vortices; 9. Simple fields of flow obtained by superposition; 10. The forces on a body in a uniform stream; 11. Simple fields of flow obtained by conformal mapping; 12. Discontinuous flow; 13. The design of wing profiles; 14. Point sources and doublets in axi-symmetric flow; 15. Slender-body theory.

In the preface the author states: 'The object of this book is to provide an introduction to Fluid Dynamics, primarily for students reading for Honours in Mathematics and Theoretical Physics. There is an undoubted need for such a work, for although we have the comprehensive treatises of Lamb and of Milne-Thomson, there is no elementary account of the basic principles of modern Fluid Dynamics. The works of Basset and Ramsey still repay consultation, but the emphasis in Fluid Dynamics is no longer on analytical solutions of ingenious problems but on the development of the physical significance of the fundamental principles.

'Our main purpose therefore is to introduce Fluid Dynamics as a branch of dynamics and to concentrate on the fundamental dynamical principles and their immediate applications to the types of fluid flow which are actually observed or produced, especially to the "disturbance" flow produced by the motion of a solid body through a fluid....

'This book, however, is limited to the theoretical aspect of Fluid Dynamics, and is, moreover, restricted to the theory of the perfect fluid....'

Accordingly, readers will expect an account of the scope and physical limitations of the frictionless perfect-fluid model, a difficult and interesting subject which the author would be well qualified to treat. The perfect fluid was discredited early in the history of hydrodynamics when it was found to lead to D'Alembert's paradox, that bluff bodies in steady motion through a perfect fluid experience no drag. Since then, however, the perfect fluid has been shown to be applicable in many important situations (not including the steady motion of a bluff body), in particular in the theory of aerofoils and slender bodies, and of water and sound waves. To explain the reasons for this success must involve some discussion of such topics as viscosity, boundary layers, flow separation, hydrodynamic stability, etc., which in the older books were relegated to the later chapters, but without which no real understanding of the subject is possible.

Such an explanation is not to be found in Prof. Temple's book. Viscosity and the Reynolds number are each mentioned once (on p. 20), the boundary layer a few times, and no comparison with experiments is given at any point. A student without previous knowledge of fluid dynamics would necessarily get a very erroneous idea of its problems and methods from this book. For instance, when the potential flow past a circular cylinder is discussed on p. 118, should not a warning be given that the results may not be applied to steady flow, but that they may be applied to the initial stage of flow accelerated from rest and also to oscillations of small amplitude? The emphasis throughout this book is not really more physical than in the older books mentioned by Prof. Temple. It seems a pity that we are not given a full discussion of any one problem, say the flow past a circular cylinder or the Thwaites flap mentioned in chapter 9, including all that is known experimentally, with diagrams and photographs.

Although the neglect of the non-mathematical aspects severely limits the utility of this book as a first introduction, the discussion of the mathematics of potential flow is in many places original and elegant and will appeal to the reader with some previous knowledge, and some common difficulties are clearly explained in the book.

In chapter 1 it is emphasized that Newton's laws must be applied to a mass of fluid consisting always of the same particles. In §4.7 Kelvin's circulation theorem is proved in the Lagrangian description of motion, whereby obscurities commonly found in the proof of this theorem are avoided. Chapter 8 is devoted to the representation of two-dimensional flows by sources and dipoles (Green's theorem) and also by sources and vortices. The treatment, based on complex integration is very neat; I have not seen it before. The discontinuous flow past a plate is calculated more simply than in the standard texts, by Helmholtz's original method. Many of the problems and examples are well chosen and interesting and often taken from recent research work. It is made clear, at any rate, that the subject is not now a merely academic exercise. However, the ideal well-balanced introduction to fluid dynamics still remains to be written.

F. URSELL

Mathematical Theory of Compressible Fluid Flow. By RICHARD VON MISES, completed by HILDA GEIRINGER and G. S. S. LUDFORD. New York: Academic Press Inc., 1958. 514 pp. $15.00.

Ten years and more ago, the publication of any book on the theory of compressible fluid flow was an event of great importance to all workers in the field, and whatever the merits and demerits of the work, it was almost bound to have some value. But in the past decade the situation has changed completely; an almost embarrassing number of books on this subject has appeared, and any new works must now exhibit special qualities of originality or exposition if they are to be successful. It is pleasant to be able to record that these qualities are evident in the case of this book by von Mises. This is the third volume in the Series of Monographs on Applied Mathematics and Mechanics prepared under the auspices of the Applied Physics Laboratory, The Johns Hopkins University, and is a worthy companion to its distinguished predecessors in the series. According to the preface, the first half of the book was completed by von Mises before his untimely death in 1953, and the second half was written by Hilda Geiringer (Mrs R. von Mises) and G. S. S. Ludford using von Mises's papers and lecture notes for guidance. So well has the addition been made that much more than a casual examination of the text is required to detect the change of authorship.

The book is divided into five chapters, the first two of which deal with the general theory of compressible flow. The equations of motion are derived by what might be described as classical methods, except that the usual equations of state and energy are replaced by an original feature of the author's in the form of a *specifying equation*, which is a general relation between the pressure, density, and velocity, their derivatives, and the space co-ordinates and time. Energy relations and the influence of viscosity and heat conduction in a fluid are then considered, and the first chapter concludes with a discussion of the acoustical equations. The second chapter contains derivations and discussions, often in considerable and valuable detail, of some consequences of the general equations. The theory of characteristics is introduced here; the treatment given is almost entirely mathematical (rather than physical), but is very detailed, particularly for the case of two variables, and the present reviewer found it most illuminating on a number of points.

The next chapter is concerned with one-dimensional flow, and starts with an article on steady flow with viscosity and heat conduction present, followed by articles on non-steady flow of an ideal fluid, and simple waves. Shocks are then introduced and are given a long discussion on the basis of an ideal gas with constant specific heats, with due attention to the non-isentropic flow behind curved shocks.

Chapter IV contains an account of the theory of plane steady potential flow, with emphasis on the supersonic case and the method of characteristics. The hodograph equations are derived and discussed, both for the velocity potential and stream function and for their Legendre transforms. Then the exact solutions for radial, vortex, and spiral flows are examined, and limiting lines are given a brief mention. Articles on the Chaplygin–Kármán–Tsien approximation and simple waves are followed by extensive discussions of limit and branch lines, and Chaplygin's hodograph method.

The last chapter deals with integration theory and shocks. Hodograph theory is developed still further, following the method of Lighthill and Cherry, and there is a useful account of Bergman's integral transformation method. Further aspects of shock theory and non-isentropic flows are given, and the chapter ends with a particularly interesting article on the current theoretical ideas and difficulties associated with the transonic flow problem.

The work ends with nearly forty pages of Notes and Addenda, which are collected footnotes on the main text. These notes are extraordinarily informative, and the reviewer found most of them quite fascinating, particularly for the amount of historical matter which is contained in them. It appears from the preface that these notes were placed together at the back of the volume, instead of on the actual pages to which they refer, in accord with 'von Mises's practice of keeping text free from distraction'; however, on the assumption that they are meant to be read in conjunction with the appropriate matter of the text, the continual necessity for reference to the end of the book is far more distracting.

The whole work is essentially mathematical in concept, as would be expected from such an eminent mathematician as von Mises, and personally the reviewer found the presentation and content very much to his taste. But there are other points of view on this matter, and undoubtedly there are many who would prefer a closer contact with the physical ideas behind the theory than is to be found here. As it is, the book provides a mine of information on its subject which should be invaluable to students and research workers in the field of theoretical mechanics, and engineers with mathematical leanings will also find much which will be of profit to them. It must be remarked that the book is not easy reading, in the sense that much more than a casual study is required in many places in order to extract the full value of the text; of course, this is almost inevitable in a work of any depth.

The reviewer found himself at variance with the author over the derivation of the equations of motion. The reviewer holds the opinion that the integral forms of these equations are more fundamental than their differential forms, since the former cover more general types of flow than the latter, including discontinuous motions, and the differential forms can be deduced from the integral forms once suitable conditions have been imposed on the derivatives of the dependent variables. Thus, in a fully deductive mathematical treatment, it would seem to be more appropriate to take the integral equations as a starting point. Also there are difficulties in the application of Newton's Laws of Motion which are not mentioned by the author.

The preface states that 'The present book contains no extensive discussion of the approximation theories.... It was the author's intention to discuss these in the second part of his work....' The death of the author before this second part could be written is a great loss, for it is certain that von Mises would have had much of interest to write on this topic, as on those subjects which he has covered so well in the present book.

G. N. WARD